

On the Degrees-of-freedom of the 3-user MISO Broadcast Channel with Hybrid CSIT

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Abstract—The 3-user multiple-input single-output (MISO) broadcast channel (BC) with hybrid channel state information at the transmitter (CSIT) is considered. In this framework, there is perfect and instantaneous CSIT from a subset of users and delayed CSIT from the remaining users. We present new results on the degrees of freedom (DoF) of the 3-user MISO BC with hybrid CSIT. In particular, for the case of 2 transmit antennas, we show that with perfect CSIT from one user and delayed CSIT from the remaining two users, the optimal DoF is $5/3$. For the case of 3 transmit antennas and the same hybrid CSIT setting, it is shown that a higher DoF of $9/5$ is achievable and this result improves upon the best known bound. Furthermore, with 3 transmit antennas, and the hybrid CSIT setting in which there is perfect CSIT from two users and delayed CSIT from the third one, a novel scheme is presented which achieves $9/4$ DoF. Our results also reveal new insights on how to utilize hybrid channel knowledge for multi-user scenarios.

I. INTRODUCTION

There has been a significant recent interest in understanding the impact of delayed CSIT on the DoF of multi-user MIMO systems. Maddah-Ali and Tse [1] showed that for the K -user MISO broadcast channel, with a K -antenna transmitter and K single antenna users, the optimal sum DoF is given by the elegant formula $K/(1 + \frac{1}{2} + \dots + \frac{1}{K})$. This result shows that even completely delayed CSIT can significantly increase the DoF by exploiting overheard side-information at the users/receivers. However, this result assumes homogeneity in channel knowledge in the following sense: CSIT from every user is delayed. This assumption may not always be true in practice and the delays experienced in acquiring CSIT can vary across users. Such scenarios can arise when some of the users can supply timely CSIT whereas others supply CSIT with delay (which could be a result of factors such as uplink overhead or infrequent feedback). This heterogeneity of channel knowledge motivates the framework of hybrid CSIT.

To formalize the hybrid CSIT framework, we denote the availability of CSIT from a particular receiver through a variable I_{CSIT} , which can take values either P or D. For receiver k , the state $I_{\text{CSIT}}^k = \text{P}$ indicates that it supplies perfect and instantaneous CSIT and the state $I_{\text{CSIT}}^k = \text{D}$ indicates that it supplies completely delayed CSIT. Thus, for a M -antenna transmitter and K single antenna receivers, i.e., the (M, K) MISO BC, there are a total of 2^K possible CSIT configurations. The understanding of how to optimally utilize hybrid CSIT is far from complete and optimal results are known only for the case of $(2, 2)$ MISO BC. If the transmitter has perfect CSIT from both the receivers (PP), then the optimal DoF

is 2 which can be achieved using beamforming techniques [2]. When there is delayed CSIT from both the users (DD), then the optimal DoF reduces to $4/3$ [1]. For the hybrid CSIT scenario in which the transmitter has instantaneous CSI from receiver 1 and delayed CSI from receiver 2, (hybrid CSIT: PD) it was shown in [3] that the optimal DoF is $3/2$.

We next come to the simplest non-trivial extension of the hybrid CSIT setting for more than 2 receivers, i.e., the case of three receiver, i.e. $(M, 3)$ MISO BC which is the main focus of this paper. Here, a total of $2^3 = 8$ possible CSIT configurations, namely PPP, PPD, PDP, DPP, PDD, DPD, DDP, DDD can arise. Essentially, we have 4 non-degenerate CSIT configurations, namely PPP, PDD, PPD, DDD, which depending on the number of transmit antennas ($M = 2$ or $M = 3$) lead to two scenarios.

The optimal DoF for the $(2, 3)$ MISO BC is 2 with either PPP or PPD configurations (limited by 2 transmit antennas). For the DDD configuration, it has been shown in [1] that the optimal DoF is given by $3/2$. Therefore, the only remaining case for which optimal DoF was not known prior to this work is the PDD configuration. In this paper, we show that the optimal DoF for this CSIT configuration is $5/3$. We next consider the $(3, 3)$ MISO BC with hybrid CSIT. For this setting, the optimal DoF is 3 with the PPP configuration [2] and reduces to $18/11$ in the DDD configuration [1]. However, the optimal DoF values for the remaining two CSIT configurations i.e., PDD and PPD are unknown. We present novel schemes for these two configurations which exploit hybrid channel knowledge to achieve sum DoF values of $9/5$ and $9/4$ respectively. The paper that is most relevant to this work is [8], in which outer bounds for the (K, K) MISO BC are obtained for general hybrid CSIT configurations. In addition, a coding scheme for $(3, 3)$ MISO BC with PDD configuration is given which achieves $5/3$ DoF. Our results improve upon this bound to achieve $9/5$ DoF, as well as establish that the optimal sum DoF for the $(2, 3)$ MISO BC for PDD configuration is $5/3$.

Our results show how to utilize hybrid CSIT, which is a mixture of instantaneous and outdated CSIT from different receivers. While the core idea to exploit overheard side information at those receivers supplying delayed CSIT bears similarities to [1], the new technical challenge is that this exploitation must be done so that instantaneous CSIT (from other receivers) can be simultaneously harnessed.

II. SYSTEM MODEL

A (M, K) MISO broadcast channel with M -transmit antennas and K -single antenna receivers with hybrid CSIT is considered. The received signal at the k th receiver is given by

$$y_k(t) = \mathbf{h}_k(t)\mathbf{x}(t) + z_k(t), \quad (1)$$

where $\mathbf{x}(t)$ is the $M \times 1$ channel input at time t with $E(|\mathbf{x}(t)|^2) \leq P_T$, where P_T is the average input power constraint; $\mathbf{h}_k(t)$ is the $1 \times M$ channel vector from the transmitter to receiver k at time t . Without loss of generality, $\mathbf{h}_k(t)$ is assumed to be sampled from any continuous distribution (e.g., Rayleigh) with an identity co-variance matrix, and are independent and identically distributed (i.i.d.) across time and also i.i.d. across receivers. The additive noise $z_k(t)$ is distributed according to $\mathcal{CN}(0, 1)$ for $k = 1, \dots, K$ and assumed to be independent of all other random variables. Throughout the paper, we assume the availability of global channel state information at the receivers (i.e., full CSIR).

The rate tuple (R_1, R_2, \dots, R_K) , with $R_k = \log(|W_k|)/n$, where W_k is the message intended for the k th receiver, is achievable if there exist an encoding function and K decoding functions (one for each receiver) such that the probability of decoding error at each receiver can be made arbitrarily small. The encoding function depends on the specific hybrid CSIT configuration. For example, when the transmitter has perfect and instantaneous CSIT from the 1st receiver and delayed CSIT from the remaining $(K - 1)$ receivers, the encoding function depends on the current and past CSIT of the 1st receiver and only the past CSIT of the other $(K - 1)$ receivers. For other hybrid CSIT settings, the encoding and the decoding functions can be defined similarly. In this paper, we focus on the sum DoF of the M -antenna, K -receiver MISO BC, (henceforth referred to as the (M, K) -MISO BC) which is defined as $\text{DoF}(M, K) = \lim_{P_T \rightarrow \infty} \max \sum_{k=1}^K \frac{R_k}{\log(P_T)}$, where the maximum is over all achievable K -tuples (R_1, \dots, R_K) .

III. MAIN RESULTS

Theorem 1: The optimal sum DoF of the $(2, 3)$ MISO BC with instantaneous CSIT from receiver 1 and delayed CSIT from receivers 2, 3 i.e., $(I_{\text{CSIT}}^1, I_{\text{CSIT}}^2, I_{\text{CSIT}}^3) = \text{PDD}$ is

$$\text{DoF}^{\text{PDD}}(2, 3) = \frac{5}{3}. \quad (2)$$

Theorem 2: The sum DoF of the $(3, 3)$ MISO BC with instantaneous CSIT from receiver 1 and delayed CSIT from receivers 2, 3 i.e., $(I_{\text{CSIT}}^1, I_{\text{CSIT}}^2, I_{\text{CSIT}}^3) = \text{PDD}$ satisfies

$$\frac{9}{5} \leq \text{DoF}^{\text{PDD}}(3, 3) \leq \frac{17}{9}. \quad (3)$$

Theorem 3: The sum DoF of the $(3, 3)$ MISO BC with instantaneous CSIT from receivers 1 and 2, and delayed CSIT from receiver 3 i.e., $(I_{\text{CSIT}}^1, I_{\text{CSIT}}^2, I_{\text{CSIT}}^3) = \text{PPD}$ satisfies

$$\frac{9}{4} \leq \text{DoF}^{\text{PPD}}(3, 3) \leq \frac{7}{3}. \quad (4)$$

The converse proofs (upper bounds) follow directly from the arguments in [1], [8], [9] and are therefore omitted.

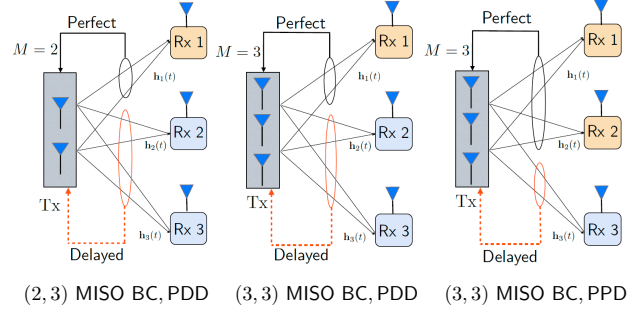


Fig. 1: $(M, 3)$ MISO BC with Hybrid CSIT.

Henceforth, we present new achievable schemes (lower bounds on DoF) which are the main contributions of this paper.

IV. ACHIEVABILITY PROOFS

We first refresh the optimal scheme for the $(2, 2)$ MISO BC [3] with hybrid CSIT configuration PD (instantaneous CSIT from user 1, delayed CSIT from user 2) that achieves the DoF value of $3/2$. Here, the transmitter sends two symbols (a_1, a_2) to user 1 and one symbol b to user 2 in two time slots as follows: in the first time slot, it sends $[a_1 \ a_2]^T + \mathbf{h}_1(1)^\perp [b \ 0]^T$, where $\mathbf{h}_1(1)^\perp$ denotes a 2×2 projection operator orthogonal to $\mathbf{h}_1(1)$. User 1 receives a linear combination $\mathcal{L}_1(a_1, a_2)$, whereas user 2 gets a linear combination of (a_1, a_2) and b , denoted by $\mathcal{L}_2(a_1, a_2) + b$. Thus, $\mathcal{L}_2(a_1, a_2)$ is a symbol which is desirable by both users since user 1 can decode (a_1, a_2) from $\mathcal{L}_1(a_1, a_2)$, $\mathcal{L}_2(a_1, a_2)$, whereas user 2 can use it to decode b . Thus, we call $\mathcal{L}_2(a_1, a_2)$ as an *order 2 symbol*, i.e., a symbol desired by 2 users; this symbol can be sent in the second time slot achieving $3/2$ DoF.

A. Order 2 DoF: $(2, 3)$ MISO BC (PDD)

While the order 2 DoF in the $(2, 2)$ MISO BC is 1 (one order 2 symbol can be delivered to two receivers in one time slot), one can do better when we consider the extension to the 3-user MISO BC. For the 3-user MISO BC, there are 3 possible types of order 2 symbols, namely, symbols desired by receivers (1, 2), symbols desired by receivers (1, 3), and symbols desired by receivers (2, 3). We first present the optimal degrees of freedom region for delivering order 2 symbols with hybrid CSIT. This result forms the basis for establishing the achievability proofs for Theorems 1 and 2.

Lemma 1: The order 2 DoF region for the $(2, 3)$ MISO BC with PDD CSIT configuration is given by

$$d_{12} + d_{13} \leq 1 \quad (5)$$

$$2(d_{12} + d_{23}) + d_{13} \leq 2 \quad (6)$$

$$d_{12} + 2(d_{23} + d_{13}) \leq 2. \quad (7)$$

Proof: The converse proof follows from the arguments similar in [1], [8], [9] and is therefore omitted. From (5)-(7), the optimal order 2 sum DoF is $5/4$ corresponding to the tuple $(d_{12}, d_{23}, d_{13}) = (\frac{1}{2}, \frac{1}{4}, \frac{1}{2})$. We next present a novel coding scheme which achieves this tuple. To this end, we denote ab, bc and ac -symbols as the order 2 symbols desired by receivers

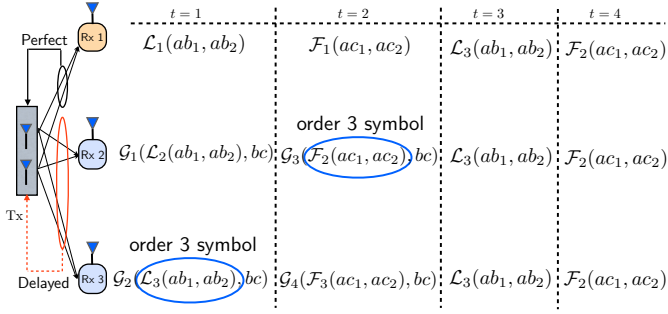


Fig. 2: Achieving 5/4 order 2 DoF; (2, 3) MISO BC.

(1, 2), (2, 3) and (1, 3) respectively and present a scheme which sends $(ab_1, ab_2, ac_1, ac_2, bc)$ to the corresponding receivers in 4 time slots. This scheme is shown in Fig. 2 and is described next:

- At $t = 1$, transmitter sends $\{ab_1, ab_2\}$ along with bc in a direction orthogonal to $\mathbf{h}_1(1)$ as $\mathbf{x}(1) = [ab_1 \ ab_2]^T + \mathbf{h}_1^\perp(1)[bc \ 0]^T$. The receiver outputs are shown in Fig. 2, where $\mathcal{G}_i(\mathcal{L}_j(ab_1, ab_2), bc)$ indicates a linear combination of $\mathcal{L}_j(ab_1, ab_2)$ and bc . At the end of $t = 1$, we note that the linear combination $\mathcal{L}_3(ab_1, ab_2)$ is useful for all the 3 receivers, i.e., this is an order 3 symbol.
- At $t = 2$, an identical scenario is created with the ac symbols by sending $\mathbf{x}(2) = [ac_1 \ ac_2]^T + \mathbf{h}_1^\perp(2)[bc \ 0]^T$. The corresponding outputs at the receivers are shown in Fig. 2, and similar to $\mathcal{L}_3(ab_1, ab_2)$, $\mathcal{F}_2(ac_1, ac_2)$ is also an order 3 symbol.
- We now note that these order 3 symbols, i.e., $\mathcal{L}_3(ab_1, ab_2)$ and $\mathcal{F}_2(ac_1, ac_2)$ can be reconstructed at the transmitter via delayed CSIT from users 2 and 3. These symbols can be subsequently delivered in time slots $t = 3$ and $t = 4$ (as order 3 DoF for a 3-receiver MISO BC is 1).

Finally, at the end of these 4 time slots, the 1st receiver can decode ab_1, ab_2 using $\mathcal{L}_1(ab_1, ab_2), \mathcal{L}_3(ab_1, ab_2)$ and ac_1, ac_2 using $\mathcal{F}_1(ac_1, ac_2), \mathcal{F}_2(ac_1, ac_2)$. The 2nd receiver can decode bc by canceling the interference from $\mathcal{F}_2(ac_1, ac_2)$. This bc symbol is used to recover $\mathcal{L}_2(ab_1, ab_2)$ from $\mathcal{G}_1(\mathcal{L}_2(ab_1, ab_2), bc)$. Thus using $\mathcal{L}_2(ab_1, ab_2), \mathcal{L}_3(ab_1, ab_2)$, it can reconstruct the symbols ab_1, ab_2 by solving two LCs of two symbols. Along similar lines, the 3rd receiver can reconstruct (ac_1, ac_2, bc) . Thus, the order 2 DoF tuple $(d_{12}, d_{23}, d_{13}) = (\frac{1}{2}, \frac{1}{4}, \frac{1}{2})$ is achievable for (2, 3) MISO BC.

B. Theorem 1: 5/3 DoF – (2, 3) MISO BC (PDD)

We next present the optimal scheme that uses the result of Lemma 1 to achieve the DoF tuple $(d_1, d_2, d_3) = (1, 1/3, 1/3)$. Specifically, the scheme sends 12 symbols to the 1st receiver and 4 symbols each to receivers 2 and 3 (denoted by $\{a_i\}_{i=1}^{12}, \{b_i\}_{i=1}^4$ and $\{c_i\}_{i=1}^4$) in 12 time slots.

Stage 1 – generating order 2 symbols: This stage consists of 3 phases, denoted by phase- bc , phase- ab and phase- ac . Phase- bc corresponds to creating one order 2 symbol for receivers (2, 3). Phase- ab and phase- ac are used to create $\{ab_1, ab_2\}$ and $\{ac_1, ac_2\}$ respectively. Fig. 3 shows the

outputs at the receivers in this stage and the mechanism of generating order 2 symbols.

Phase- bc : creating 1 bc symbol

- At $t = 1$, the transmitted and received signals are given by

$$\mathbf{x}(1) = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \mathbf{h}_1^\perp(1) \begin{bmatrix} b_1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} y_1(1) \\ y_2(1) \\ y_3(1) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_1 \\ \mathcal{L}_1(\mathcal{A}_2, b_1) \\ \mathcal{L}_2(\mathcal{A}_{1,2}, b_1) \end{bmatrix}. \quad (8)$$

At the end of $t = 1$, notice that \mathcal{A}_2 is useful for both 1st and 2nd receivers. Since the transmitter has delayed CSI from the 2nd receiver, it can reconstruct \mathcal{A}_2 . Note that $\mathcal{A}_1, \mathcal{A}_2$ and $\mathcal{A}_{1,2}$ are linear combinations of the two symbols (a_1, a_2) .

- At $t = 2$, the transmitter sends \mathcal{A}_2 along with a new symbol b_2 as $\mathbf{x}(2) = [\mathcal{A}_2 \ 0]^T + \mathbf{h}_1^\perp(2)[b_2 \ 0]^T$ and the outputs at the receivers are shown in Fig. 3. At the end of $t = 2$, the 1st receiver can decode the symbols a_1, a_2 using two linearly independent combinations \mathcal{A}_1 and \mathcal{A}_2 . $\mathcal{L}_4(\mathcal{A}_2, b_2)$ (with receiver 3) is useful for the 2nd receiver as it helps decode b_1, b_2 (since it can recover 3 symbols \mathcal{A}_2, b_1, b_2 from $\mathcal{L}_1, \mathcal{L}_3$ and \mathcal{L}_4). Thus, $\mathcal{L}_4(\mathcal{A}_2, b_2)$ is an ingredient for creating the bc -symbol.

- At $t = 3$ and $t = 4$, the transmitter sends new symbols a_3, a_4 for the 1st receiver along with c_1, c_2 for the 3rd receiver as $\mathbf{x}(3) = [a_3 \ a_4]^T + \mathbf{h}_1^\perp(3)[c_1 \ 0]^T$ and $\mathbf{x}(4) = [\mathcal{A}_4 \ 0]^T + \mathbf{h}_1^\perp(4)[c_2 \ 0]^T$, and the outputs at the receivers are shown in Fig. 3. The side information $\mathcal{L}_7(\mathcal{A}_4, c_2)$ at receiver 2 is useful for the 3rd receiver (to recover c_1, c_2). Therefore from $t = 2$ and $t = 4$, $\mathcal{L}_4(\mathcal{A}_2, b_2) + \mathcal{L}_7(\mathcal{A}_4, c_2)$ is the bc symbol.

Phase- ab : creating 2 ab symbols

Here, we create 2 ab -symbols. In this regard, the transmitter sends $\{a_j\}_{j=5}^8$ to the 1st receiver and (b_3, b_4) to the 2nd receiver as $\mathbf{x}(5) = [a_5 \ a_6]^T + \mathbf{h}_1^\perp(5)[b_3 \ 0]^T$, $\mathbf{x}(6) = [a_7 \ a_8]^T + \mathbf{h}_1^\perp(6)[b_4 \ 0]^T$. It is clear from Fig. 3, that \mathcal{A}_6 and \mathcal{A}_8 are two ab -symbols useful for both the receivers 1 and 2.

Phase- ac : creating 2 ac symbols

The above phase is repeated here by sending $\mathbf{x}(7) = [a_9 \ a_{10}]^T + \mathbf{h}_1^\perp(7)[c_3 \ 0]^T$ and $\mathbf{x}(8) = [a_{11} \ a_{12}]^T + \mathbf{h}_1^\perp(8)[c_4 \ 0]^T$. It is clear from Fig. 3 that \mathcal{A}_{10} and \mathcal{A}_{12} are ac -symbols desired by receivers 1 and 3.

Summary of Stage 1: At the end of stage 1, we have 5 order 2 symbols: $(ab_1, ab_2) = (\mathcal{A}_6, \mathcal{A}_8)$, $(ac_1, ac_2) = (\mathcal{A}_{10}, \mathcal{A}_{12})$ and $bc = \mathcal{L}_4(\mathcal{A}_2, b_2) + \mathcal{L}_7(\mathcal{A}_4, c_2)$ that can be delivered in stage 2.

Stage 2 – delivering order 2 symbols: The 5 order 2 symbols created in stage 1 can be delivered in 4 time slots using the scheme developed in Lemma 1. Upon receiving these order 2 symbols, all receivers can decode their desired symbols. For example, upon receiving \mathcal{A}_6 , the 1st receiver can decode a_5, a_6 via using \mathcal{A}_5 which was already received at $t = 5$. The 2nd receiver can use \mathcal{A}_6 to cancel interference in the symbol $\mathcal{L}_9(\mathcal{A}_6, b_3)$ and decode b_3 . Similar reasoning holds true for the other symbols at the receivers. Overall, the transmitter spent 8 time slots in the 1st stage and 4 time slots in the 2nd stage, which gives the optimal DoF tuple $(\frac{12}{12}, \frac{4}{12}, \frac{4}{12}) = (1, \frac{1}{3}, \frac{1}{3})$ i.e., $\text{DoF}^{\text{PDD}}(2, 3) = 5/3$.

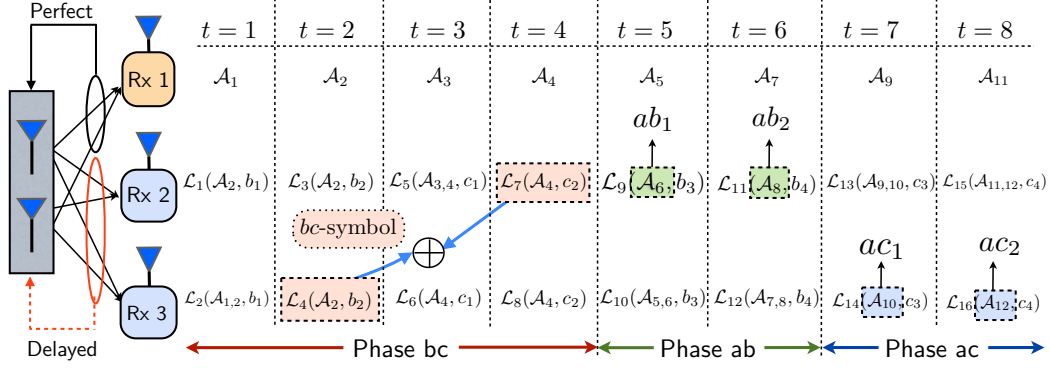


Fig. 3: Generating order 2 symbols – (2, 3) MISO BC, CSIT configuration PDD.

C. Theorem 2: 9/5 DoF – (3, 3) MISO BC (PDD)

In this section, we present a scheme that achieves the tuple $(d_1, d_2, d_3) = (1, \frac{2}{5}, \frac{2}{5})$, i.e., total of 9/5 DoF and improves upon the best known bound of 5/3 [8]. We present a scheme that sends 10 symbols to the 1st receiver and 4 symbols each to receivers 2 and 3 in a total of 10 time slots.

Remark 1: Similar to the scheme for (2, 3) MISO BC, this scheme also has two stages, stage 1 dedicated to generating order 2 symbols and stage 2 for their delivery using Lemma 1. However, there are two key distinctions – a) the mechanism of generating order 2 symbols is different, and b) the rate of creation of order 2 symbols is higher for (3, 3) MISO BC, leading to a higher DoF value of 9/5 compared to 5/3.

Stage 1 – generating order 2 symbols: This stage (shown in Fig. 4) is split into two distinct phases: Phase-*bc* takes 3 time slots and generates 1 *bc*-symbol and Phase-(*ab, ac*) takes 3 time slots to jointly generate 2 *ab*-symbols and 2 *ac*-symbols.

Phase-bc: creating 1 bc-symbol

This phase sends (a_1, a_2, a_3) along with (b_1, b_2) and (c_1, c_2) in three time slots as follows:

$$\begin{aligned} \mathbf{x}(1) &= \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \rightarrow \begin{bmatrix} y_1(1) \\ y_2(1) \\ y_3(1) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \end{bmatrix}, \\ \mathbf{x}(2) &= \begin{bmatrix} 0 \\ \mathcal{A}_2 \\ 0 \end{bmatrix} + \mathbf{h}_1^\perp(2) \begin{bmatrix} 0 \\ b_1 \\ b_2 \end{bmatrix} \rightarrow \begin{bmatrix} y_1(2) \\ y_2(2) \\ y_3(2) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_2 \\ \mathcal{L}_1(\mathcal{A}_2, \mathcal{B}_1) \\ \mathcal{L}_2(\mathcal{A}_2, \mathcal{B}_2) \end{bmatrix}, \\ \mathbf{x}(3) &= \begin{bmatrix} 0 \\ 0 \\ \mathcal{A}_3 \end{bmatrix} + \mathbf{h}_1^\perp(3) \begin{bmatrix} 0 \\ c_1 \\ c_2 \end{bmatrix} \rightarrow \begin{bmatrix} y_1(3) \\ y_2(3) \\ y_3(3) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_3 \\ \mathcal{G}_1(\mathcal{A}_3, \mathcal{C}_1) \\ \mathcal{G}_2(\mathcal{A}_3, \mathcal{C}_2) \end{bmatrix}. \end{aligned}$$

It is clear that at the end of this phase, receiver 1 is able to decode (a_1, a_2, a_3) and the transmitter can create the following *bc*-symbol that is useful for both the receivers 2 and 3:

$$bc = \mathcal{L}_2(\mathcal{A}_2, \mathcal{B}_2) + \mathcal{G}_1(\mathcal{A}_3, \mathcal{C}_1).$$

Phase-(ab, ac): creating 2 ab-symbols and 2 ac-symbols

This phase sends $\{a_j\}_{j=4}^{10}$ for receiver 1, (b_3, b_4) for receiver 2 and (c_3, c_4) for receiver 3 to generate 2 *ab* and 2 *ac*-symbols in 3 time slots (at a higher rate in comparison to Theorem 1).

• At $t = 4, 5$, the transmitter sends $\mathbf{x}(4) = [a_4 \ a_5 \ a_6]^T + \mathbf{h}_1^\perp(4)[0 \ b_3 \ b_4]^T$ and $\mathbf{x}(5) = [a_7 \ a_8 \ a_9]^T + \mathbf{h}_1^\perp(5)[0 \ c_3 \ c_4]^T$. From Fig. 4, note that $\mathcal{L}_4(\mathcal{A}_6, \mathcal{B}_4)$ (at user 3) and $\mathcal{G}_3(\mathcal{A}_8, \mathcal{C}_3)$ (at user 2) are useful for users 2 and 3 respectively.

• Thus at $t = 6$, the transmitter sends these side information symbols along with a_{10} , a new symbol for receiver 1 as $\mathbf{x}(6) = [a_{10} \ 0 \ 0]^T + \mathbf{h}_1^\perp(6)[0 \ \mathcal{L}_4(\mathcal{A}_6, \mathcal{B}_4) \ \mathcal{G}_3(\mathcal{A}_8, \mathcal{C}_3)]^T$.

We next note that in Phase-(*ab, ac*), receiver 1 has obtained a total of 3 interference free symbols and requires 4 more useful symbols in order to decode $\{a_j\}_{j=4}^{10}$. Receiver 2 uses $y_2(5)$ and $y_2(6)$ to eliminate $\mathcal{G}_3(\mathcal{A}_8, \mathcal{C}_3)$ to obtain a LC of (a_{10}, \mathcal{A}_6) and \mathcal{B}_4 denoted as $\mathcal{A}'_6 + \mathcal{B}_4$. Receiver 2 also obtains $\mathcal{L}_3(\mathcal{A}_5, \mathcal{B}_3)$ at $t = 4$ which leads to the generation of 2 *ab*-symbols: \mathcal{A}_5 and \mathcal{A}'_6 . Receiver 3 uses $y_3(4)$ and $y_3(6)$ to eliminate $\mathcal{L}_4(\mathcal{A}_6, \mathcal{B}_4)$ to obtain a LC of (a_{10}, \mathcal{A}_8) and \mathcal{C}_3 denoted as $\mathcal{A}'_8 + \mathcal{C}_3$. Thus \mathcal{A}_9 and \mathcal{A}'_8 are 2 *ac* symbols.

Stage 2 – delivering order 2 symbols: This stage delivers the 5 order 2 symbols created in stage 1, i.e., $(ab_1, ab_2, ac_1, ac_2, bc)$ in 4 time slots using the transmission scheme of Lemma 1 (also a valid scheme for the (3, 3) MISO BC). Overall we delivered 10 symbols to the 1st receiver, 4 symbols each to the 2nd and 3rd receivers in 10 time slots. Thus the DoF achieved by this scheme is given by

$$\text{DoF}^{\text{PDD}}(3, 3) \geq \frac{10 + 4 + 4}{3 + 3 + 4} = \frac{9}{5}, \quad (9)$$

i.e., the DoF triplet $(\frac{10}{10}, \frac{4}{10}, \frac{4}{10}) = (1, \frac{2}{5}, \frac{2}{5})$ is achieved.

D. Theorem 3: 9/4 DoF – (3, 3) MISO BC (PPD)

We present a scheme (shown in Fig. 5) that achieves the DoF triplet $(d_1, d_2, d_3) = (1, 1, \frac{1}{4})$ i.e., DoF of 9/4 in the PPD configuration for the (3, 3) MISO BC. In the PPD configuration, the transmitter has instantaneous CSIT from the receivers 1, 2 and delayed CSIT from receiver 3.

At $t = 1$, we send 2 symbols each to the 1st and 2nd receivers (a_1, a_2) , (b_1, b_2) in directions orthogonal to the $\mathbf{h}_2(1)$ and $\mathbf{h}_1(1)$ respectively so as to not create cross interference. Additionally we transmit one symbol, c for the 3rd receiver by using a projection operator $\mathbf{h}_{(1,2)}^\perp(1)$ such that $\mathbf{h}_1(1)\mathbf{h}_{(1,2)}^\perp(1) = \mathbf{h}_2(1)\mathbf{h}_{(1,2)}^\perp(1) = 0$. The transmitted signal is given by

$$\mathbf{x}(1) = \mathbf{h}_2^\perp(1) \begin{bmatrix} a_1 \\ a_2 \\ 0 \end{bmatrix} + \mathbf{h}_1^\perp(1) \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix} + \mathbf{h}_{(1,2)}^\perp(1) \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix}. \quad (10)$$

This is repeated again at $t = 2$ with new symbols (a_3, a_4) and

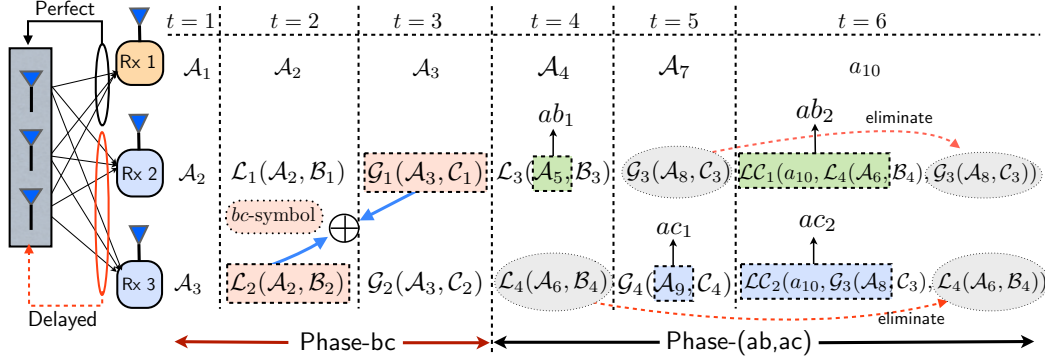


Fig. 4: Generating order 2 symbols – (3, 3) MISO BC, CSIT configuration PDD.

(b_3, b_4) for 1st and 2nd receivers but with the same c symbol for the 3rd receiver as

$$\mathbf{x}(2) = \mathbf{h}_2^\perp(2) \begin{bmatrix} a_3 \\ a_4 \\ 0 \end{bmatrix} + \mathbf{h}_1^\perp(2) \begin{bmatrix} b_3 \\ b_4 \\ 0 \end{bmatrix} + \mathbf{h}_{(1,2)}^\perp(1) \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix}. \quad (11)$$

At the end of these two time slots, notice from Fig. 5 that receiver 1 requires LCs \mathcal{A}_2 and \mathcal{A}_4 . These symbols are also useful to the 3rd receiver as it helps cancel interference and thereby decode symbol c . Similarly, \mathcal{B}_2 and \mathcal{B}_4 are required at the 2nd and 3rd receivers. Thus the goal of the next two time slots is to send these symbols in an efficient manner to the receivers. Using delayed CSIT from the 3rd receiver, transmitter can reconstruct $\mathcal{A}_2, \mathcal{A}_4, \mathcal{B}_2$ and \mathcal{B}_4 .

At $t = 3$, the transmitter sends the symbols \mathcal{A}_2 and \mathcal{B}_4 as

$$\mathbf{x}(3) = \mathbf{h}_2^\perp(3)[\mathcal{A}_2 \ 0 \ 0]^T + \mathbf{h}_1^\perp(3)[\mathcal{B}_4 \ 0 \ 0]^T$$

and the outputs at the receivers are shown in Fig. 5. Before we proceed further, let us summarize the transmission scheme until $t = 3$. The 1st receiver has LCs $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$. The 2nd receiver has the symbols $\mathcal{B}_1, \mathcal{B}_3, \mathcal{B}_4$ and the 3rd receiver has $\mathcal{L}_1(\mathcal{A}_2, \mathcal{B}_2, c)$, $\mathcal{L}_2(\mathcal{A}_4, \mathcal{B}_4, c)$ and $\mathcal{L}_3(\mathcal{A}_2, \mathcal{B}_4)$. The 1st and 2nd receivers need \mathcal{A}_4 and \mathcal{B}_2 respectively as it helps them decode their desired symbols. Hence, after $t = 3$, receiver 3 can:

- eliminate \mathcal{A}_2 from $(\mathcal{L}_1(\mathcal{A}_2, \mathcal{B}_2, c), \mathcal{L}_3(\mathcal{A}_2, \mathcal{B}_4))$ to form $\mathcal{L}_4(\mathcal{G}_1(\mathcal{B}_2, \mathcal{B}_4), c)$; and
- eliminate \mathcal{B}_4 from $(\mathcal{L}_2(\mathcal{A}_4, \mathcal{B}_4, c), \mathcal{L}_3(\mathcal{A}_2, \mathcal{B}_4))$ to form $\mathcal{L}_5(\mathcal{G}_2(\mathcal{A}_2, \mathcal{A}_4), c)$.

At $t = 4$, the transmitter sends

$$\mathbf{x}(4) = \mathbf{h}_2^\perp(4)[\mathcal{G}_2(\mathcal{A}_2, \mathcal{A}_4) \ 0 \ 0]^T + \mathbf{h}_1^\perp(4)[\mathcal{G}_1(\mathcal{B}_2, \mathcal{B}_4) \ 0 \ 0]^T,$$

From Fig. 5, it is clear that receivers 1 and 2 can decode (a_1, a_2, a_3, a_4) , and (b_1, b_2, b_3, b_4) respectively. Receiver 3 can decode the symbol c from $\mathcal{L}_4(\mathcal{G}_1(\mathcal{B}_2, \mathcal{B}_4), c)$, $\mathcal{L}_5(\mathcal{G}_2(\mathcal{A}_2, \mathcal{A}_4), c)$ and $\mathcal{L}_6(\mathcal{G}_1(\mathcal{B}_2, \mathcal{B}_4), \mathcal{G}_2(\mathcal{A}_2, \mathcal{A}_4))$. In summary, this scheme achieves the DoF triplet $(1, 1, \frac{1}{4})$ or in other words the sum DoF of $\frac{9}{4}$. This concludes the achievability proof for Theorem 3.

V. CONCLUSIONS

In this paper, we investigated the impact of hybrid CSIT on the degrees-of-freedom of 3-receiver MISO BC. Novel achievable schemes were presented for various hybrid CSIT

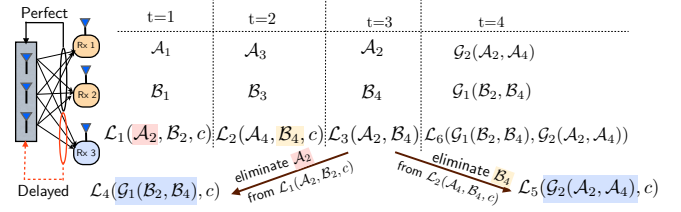


Fig. 5: Achieving 9/4 DoF for (3, 3) MISO BC with PPD.

configurations which established the optimal DoF (for the (2, 3) MISO BC) and improved upon the best known achievable DoF (for the (3, 3) MISO BC). Our results show that an important aspect when dealing with hybrid CSIT is the generation and transmission of higher order symbols which are desired by multiple receivers. As our schemes show, this problem is far from trivial even for the 3-receiver broadcast channel. Showing the optimality of these schemes, and extensions of these ideas to more than $K = 3$ receivers are interesting open problems for future work.

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